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Non-destructive characterisation of elastic parameters of 3D printed plates

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ABSTRACT: The process of additive manufacturing (3D printing) of plates exhibits a strong analogy to the production of laminated composites. Both structures are treated as multi-layered composites, where each layer's mechanical properties can be distinct yet integrated into a cohesive structure. This paper demonstrates the ability of Classical Laminated Plate Theory (CLPT) for characterising linear engineering constants and predicting the modal response of 3D printed plates. The study explores a non-destructive inverse technique based on dynamic tests. By employing non-invasive vibration tests, numerical modelling (SIMULIA/ABAQUS), and direct optimization processes (SIMULIA/Isight), the elastic parameters of individual printed layers are identified. The characterization of engineering constants is presented on a thin 3D printed plate with a symmetric layout of layers. The results are further validated by dynamic response prediction for a second symmetric plate with different printing paths. This research indicates that CLPT can be used for linear dynamic response prediction of thin 3D printed plates.

KEYWORDS: vibration test, 3D printing, inverse technique, FEM, engineering constants

1. Introduction

The layered method of producing 3D printed plates is analogous to the process of fabricating fibre-reinforced laminates. The application of a single filament path (line) in 3D printing corresponds directly to the process of laying continuous fibres in laminates. Furthermore, both processes allow precise control over the direction of material application in subsequent layers, enabling tailored anisotropic properties in the final structure. This ability to manipulate the orientation and arrangement of each layer is crucial for optimising mechanical performance and achieving desired structural characteristics in both 3D printed and laminated composite materials. The above motivates the application of laminated plates theories for mechanical and dynamic response of 3D printed plates [1-3]. The aim of the current study is to characterise engineering constants of an individual layer of a 3D printed thin plate. The proposed approach engages a non-destructive inverse technique based on dynamic tests, numerical modelling (FEM), and direct optimization processes. The classical laminated plate theory (CLPT) is used for numerical modelling of a plate. The results are successfully validated by modal response prediction for another 3D printed plate.

2. Materials and Methods

3D printed thin plates are consider on the current study. Assuming the analogy to the laminated composites, the fibre orientation (φ) of an individual layer (ply) corresponds to the filament path of the printed layer of a plate. The plates were printed using polyamide filament strengthen with short carbon fibres (CF-PA-12). The individual layer of a plate is orthotropic in the plane stress state. Then, there are four in-plane independent engineering constants to be characterised: E_1 , E_2 , G_{12} , υ_{12}. Two types of plates were developed – Plate 1 with a stacking sequence $[90_3/0_6]_s$ and dimensions (a x b x h mm – fig. 1) 150 x 100 x 3.6 mm, and Plate 2 with a stacking sequence $[90_3/-45/45/-45_2/45/-45]_s$ and dimensions 150 x 100 x 3.6 mm. Plate 1 was used for the characterisation purpose, and Plate 2 was used for the results validation. For the statistics purpose, 5 samples for each of Plate 1 and Plate 2 were printed.

2.1 The inverse technique

The inverse technique based on vibration tests and direct optimisation is used in the present study (fig. 2). The procedure begins with selection of initial values (trial parameters) for elastic constants (parameters to be identified) in order to start the FEM analysis of a plate (eigenfrequencies extraction). In parallel, a vibration test is carried out in order to extract experimental resonant frequencies. Next, the optimisation of optimal engineering constants is performed (SIMULIA/Isight) by minimising the objective function (a relative error between the experimental and the numerical data):

$$\theta(\mathbf{x}) = \sum_{n=1}^{I} \left[\frac{\left(f_n^{EXP}\right)^2 - \left(f_n^{FEM}\right)^2}{\left(f_n^{EXP}\right)^2}\right]^2 \qquad (1)$$

where f_n^{EXP} are the experimentally determined resonant frequencies; f_n^{FEM} are the numerically calculated eigenfrequencies; $\mathbf{x}$ is the vector of unknown parameters; I is the total number of eigenfrequencies considered in the analysis. To minimise the objective function, the following optimisation problem was solved: $min\theta(\mathbf{x})$, subjected to

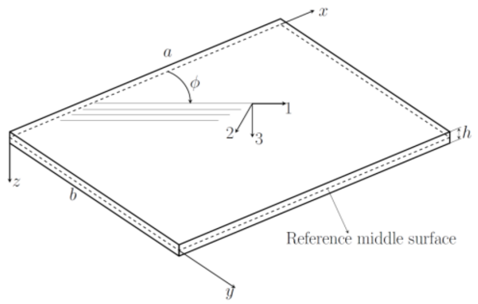

Fig. 1. Geometrical characteristics of a 3D printed plate

constraints $\mathbf{x}_i^L < \mathbf{x}_i < \mathbf{x}_i^U$, where $\mathbf{x}$=[$\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, $\mathbf{x}_4$]=[E_1, E_2, G_{12}, υ_{12}], $\mathbf{x}_i^L$ and $\mathbf{x}_i^U$ are lower and upper limits of the identification parameters, respectively. The Hooke-Jeeves optimisation method was used to find an optimal unknown vector [$\mathbf{x}$].

2.2 Numerical model

A global Cartesian coordinate system (x, y, z) was located at the middle surface of the plate as given on fig. 1. For each layer a lamina coordinate system (1, 2, 3) was defined with the direction 1 along the filament path of a lamina. A lamination angle (φ) of a single layer was defined between the x-axis and the 1-axis. The finite element (FE) method was used to develop a numerical model of the laminated plate. The SR4 layered finite elements were used with orthotropic elastic material model in SIMULIA/ABAQUS. The FE model is based on the classical lamination plate theory (CLPT), with the following displacement field:

$$u = u_0 - z\alpha,\ v = v_0 - z\beta,\ w = w_0 \qquad (3)$$

where (u_0, v_0, w_0) are the displacement components along the (x, y, z) coordinate directions, respectively, of a point on a middle surface ($z = 0$), $\alpha = \frac{\partial w_0}{\partial x}$, $\beta = \frac{\partial w_0}{\partial y}$.

2.3 Experimental analysis

Experimental modal analysis was performed using the optical vibrometer (the POLYTEC Scanning Laser Vibrometer PSV-400-B). The experimental set-up is presented on fig. 3. The plates were excited by a loudspeaker with the periodic chirp signal generated by the internal generator within the range of 0...1100 Hz. Free-free boundary conditions were provided by hanging the plates on two cotton threads. The measurements resulted with the Frequency Response Functions (FRFs), which were further used for estimation of resonant frequencies and corresponding mode shapes.

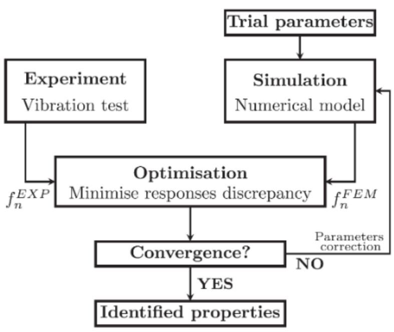

Fig. 2. Inverse technique procedure

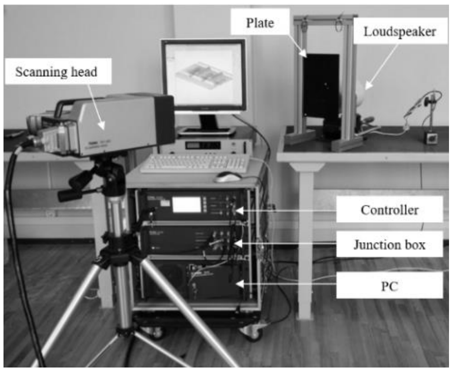

Fig. 3. Experimental set-up

2.4 Identification example

The Plate 1 was used for the identification of four engineering constants. The filament mass density was known as ρ=1030 kg/m^3. The following borders of domain for the engineering constants were used: 7.5 < E_1 < 9.9 GPa, 1.5 < E_2 < 2.5 GPa, 0.75 < G_{12} < 0.96 GPa, 0.4 < υ_{12} < 0.5. The initial unknown vector was set as: $\mathbf{x}$=[E_1,E_2,G_{12},υ_{12}]=[8.5, 2.0, 0.9, 0.42]. Having acquired experimental and numerical modal parameters, the objective function was built according to eq. 1, and the optimization was run giving the optimal unknown vector $\mathbf{x}$ = [E_1,E_2,G_{12},υ_{12}] = [9.75, 1.81, 0.82, 0.48]. The obtained material properties were then used to calculate eigenfrequencies (first 5) of the Plate 2 which were compared with the corresponding experimental resonances (tab. 1). The relative errors (Δ%) between experimental and numerical results were calculated and listed in tab. 1.

Tab. 1. Identification and validation results

Mode (n)	EXP*1	FEM1	Δ[%]	EXP*2	FEM2	Δ[%]
1	223.38	223.69	0.14	243.13	244.55	0.59
2	332.97	332.92	0.02	277.48	282.29	1.73
3	562.29	562.98	0.12	603.82	609.15	0.88
4	905.25	907.26	0.22	690.90	695.82	0.71
5	998.03	1001.00	0.30	1008.80	1030.00	2.10

1for Plate 1; 2for Plate 2; *average value from 5 plates

3. Conclusions

The layered nature of 3D printed plates is effectively modelled using CLPT, providing a robust theoretical framework for predicting their linear dynamic response. The identified engineering constants can be used for prediction plate dynamic plate's response having different printing paths of an individual layer.

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